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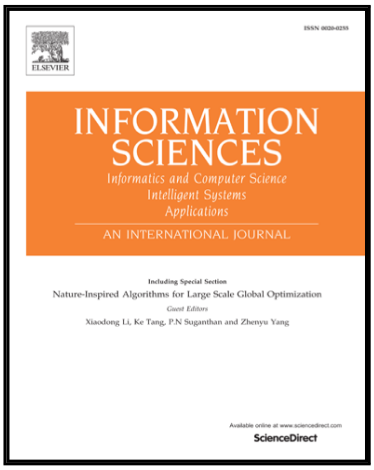

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Event-triggered dynamic output feedback control for networked control systems with probabilistic nonlinearities

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Abstract

This paper investigates the problem of dynamic output feedback control for networked nonlinear systems. A novel event-triggered mechanism (ETM) is proposed, in which a new event-triggering condition is introduced. Compared with some existing ETMs, the proposed ETM has at least three merits: i) The data-releasing rate can be further decreased, leading to a reduction of network loads; ii) During the time when the system is disturbed by external signals, this ETM can release more sampled data to the controller so that some better closed-loop performance can be achieved; and iii) Wrong decision-making on data-releasing can be avoided due to a new definition of the related error. By employing the Lyapunov-Krasovskii functional method, sufficient conditions are derived to design both controller gains and ETM parameters. An example with four cases is given to show the effectiveness and superiority of the proposed method.

Keywords: Event-triggered mechanism; Dynamic output feedback control; Probabilistic nonlinearities

1. Introduction

Over the last few decades, event-triggered mechanisms (ETMs) for networked control systems (NCSs) have received increasing interest of researchers [1, 19, 26, 34]. Different from the conventional time-triggered mechanism, data-releasing under an ETM is dependent on a predefined event-triggering condition rather than on the lapse of a fixed time period [4, 15, 33, 35]. In this situation, a large amount of redundant sampled data occupying the network resources, such as networked bandwidth, computation and energy resources of battery-based devices, can be discarded. Therefore, under an ETM, much better performance is expected for an NCS under study since the quality of network communication can be improved significantly [11, 17, 24, 27, 40, 44].

The data-releasing device receives a command from the ETM to execute the communication task. The efficiency of data-transmission can be enhanced since this implementation is based on a certain need of the control system. Therefore, the ETM may be regarded as an alternative to the traditional time-triggered mechanism. Considerable efforts have been devoted to ETMs recently. In [6, 28], the authors proposed an ETM by two steps: the first step is to design a controller for the NCSs under an assumption that the communication network is ideal (no delay and no packet dropouts), and the second step is to design an event-triggered condition under the pre-designed controller. There are two demerits if this two-step scheme works: One is called Zeno behaviour, which may result in infinite events generated by the ETM; and the other is that the controller should be known before the ETM is designed. Nevertheless, this scheme provides a good idea to improve the quality of an NCS. In [17], a discrete event-triggered scheme (DETS) is developed, which depends on the discrete sampled data rather than the real-time signal. Thus, the inter-event time is larger than one sampled period at least, resulting in an avoidance of Zeno behaviour [42]. Moreover, the closed-loop system under a DETS can be modeled as a time-delay system, based on which the parameters of the DETS and the controller gains can be co-designed in terms of linear matrix inequalities. Consequently, A DETS

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has been widely used, for example, control design for linear systems [17, 18, 23], for nonlinear systems [20, 41], consensus of multi-agent systems [3, 10, 25, 30, 37, 43, 45], filter design [2, 9], and references therein. Notice that the event-triggering condition plays an important role in further reducing the data-releasing rate or improving the efficiency of data-releasing. An adaptive threshold of ETMs is proposed in [14, 22], where the threshold is dependent on a time-varying condition rather than a fixed constant. For the purpose of reducing the number of data-releasing (NDR) during the period when a steady state is reached, an event-triggered scheme is proposed in [21] to ensure uniform ultimate bounded stability.

Some existing ETMs mainly focus on reducing the NDR while preserving certain control performance. Generally, better control performance is expected if more sampled data are transmitted to drive the physical plant. It seems paradoxical that increasing NDR is good for control performance while bad for network performance. However, during the period when the system is disturbed by external noises, it is possible for the ETM to guarantee the data-releasing rate greater than that when the system is not disturbed. In this sense, more information on the disturbed system can be achieved for the controller design. It is expected that such a controller should make sure some better performance of an NCS. To the best of authors' knowledge, few works have been done on this issue, which motivates the current study.

Output feedback control has wide applications since full information on system states is hard to measure in practical systems. Moreover, output feedback control is more complicated in control design compared with the state feedback control. Some results on ETM-based output feedback control have been reported recently. To mention a few, static output feedback based on event-triggered control schemes is introduced for stabilization in [21, 31]; and the ETM-based dynamic output feedback control can be referred to [5, 38]. In this study, by introducing a novel ETM, a dynamic output feedback control strategy is investigated to stabilize a system with probabilistic nonlinearities. The main contributions of this study are highlighted as follows:

- A new ETM is proposed, under which the data releasing rate is relatively lower in the whole time domain than some existing ETMs, except for the period when external disturbances are imposed on the system;
- A model of system nonlinearities with a probabilistic boundary is proposed by introducing some random variables;
- Based on the proposed ETM, together with the Lyapunov-Krasovskii functional approach, several sufficient conditions are obtained to guarantee the mean square stability of the nonlinear system. These conditions can be used to design the parameters in the dynamic output feedback controller and the ETM. Comparisons are made to show the effectiveness of the proposed method.

The remainder of the paper is organized as follows. Section 2 is a problem formulation including the model of nonlinear system, ETM, and the control law developing. Section 3 presents some approaches to co-designing both the dynamic output feedback controller and ETM; In Section 4, comparison results are given by a numerical example with 4 cases to demonstrate the effectiveness of the proposed method. Section 5 concludes the paper.

2. Problem formulation

2.1. Model description

Consider the following class of continuous-time systems

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1 u(t) + \sum_{i=1}^{F} \alpha_i(t) f_i(t, x(t)) + B_2 \omega(t) \\ y(t) = C_1 x(t) \\ z(t) = C_2 x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^{m_1}$, $y(t) \in \mathbb{R}^{m_2}$ and $z(t) \in \mathbb{R}^{m_3}$ are the state, the control input, the controlled output and the measurement output, respectively. $\omega(t) \in \ell_2[0, \infty)$ is disturbance input vector, $f_i(t, x(t))$ is an unknown nonlinear disturbance for $i = 1, 2 \cdots, F$; A, B_1, B_2, C_1, and C_2 are known real matrices with appropriate dimensions; The stochastic variable $\alpha_i(t)$ satisfies that $\alpha_i(t) \in \{0, 1\}$, and $\sum_{i=1}^{F} \alpha_i(t) = 1$, where F is a positive integer.

Define a set

$$\mathscr{F}_i = \{t| \|S_{i-1}x(t)\|_2 \leq \|f_i(t, x(t))\|_2 \leq \|S_i x(t)\|_2\} \tag{2}$$

with $S_0 = 0$, where S_i is a known constant matrix, and the probability of $Prob\{t \in \mathscr{F}_i\} = \alpha_i$.

Remark 1. *The nonlinearity is widely found in practice systems. A common model of the nonlinearity is defined by a unified bounded norm. However, in most cases, the variation of the nonlinearity is non-homogeneous. In (1), the nonlinearity is divided into F sections. The bigger F is, the more statistical information should be known by a statistical way in a prior.*

It is noticed that a large number of statistical experiments is needed for a big F. For facilitate implementation, one can choose an interval of the nonlinearity with a probability α, then the probability of the rest is $1 - \alpha$, i.e. $F = 2$. Therefore, we mainly discuss this simplified situation for the follow-on study. Then, it is true that $Prob\{t \in \mathscr{F}_1\} \cup Prob\{t \in \mathscr{F}_2\} = 1$, that is, $\alpha_1 = 1 - \alpha_2 = \alpha$.

2.2. *The dynamic output feedback control law*

In this study, suppose that the controller and the sensors are connected by a wireless network [7]. The sampling period is set by h. For a convenient description, the following notations are given:

- t_k: the instants of data releasing;
- a_k: the instants of the data arriving at the actuator;
- $\mathcal{L}_k$: the intervals between a_k and a_{k+1}, i.e. $\mathcal{L}_k = [a_k, a_{k+1})$;

The dynamic output feedback control law is developed by

$$\begin{cases} \dot{x}_c(t) = A_{c1}x_c(t) + A_{c2}x_c(t_k h) + B_c y(t_k h) \\ u(t) = K x_c(t) \end{cases} \tag{3}$$

for $t \in \mathcal{L}_k$, where $x_c(t) \in \mathbb{R}^n$ is the state vector of the output feedback controller; and A_{c1}, A_{c2}, B_c and K are constant real matrices to be determined.

2.3. *Improved ETM*

Define a set

$$\mathscr{I}_{t_k} = \left\{ l | e(t)^T \Omega e(t) \leq \delta_1 y^T(t_k h)\Omega y(t_k h) + \frac{\delta_2}{2}\left[y^T(t_k h)\Omega e(t) + e^T(t)\Omega y(t_k h) \right] \right\} \tag{4}$$

where $e(t) = y(t_k h) - \bar{y}(t_k h)$; $\bar{y}(t_k h)$ is a mean value between the latest released data and the current sampled data, that is $\bar{y}(t_k h) = \frac{y(t_k h) + y(t_k h + lh)}{2}$ for $l \in \mathscr{L}_k \triangleq \{0, 1, 2, \cdots, l_M\}$; δ_1 and δ_2 are positive scalars, and $\Omega > 0$ is a weighting matrix. Then borrowing the idea from [17], the next releasing instant is determined by

$$t_{k+1}h = t_k h + (l_M + 1)h. \tag{5}$$

where $l_M = \max\limits_{l \in \mathscr{I}_{t_k}} l$.

Remark 2. *If one sets $\delta_2 = 0$ in (4), the ETM is the same as that in [17]. However, the definition of $e(t)$ is different from the conventional one (see [10, 17], and the references therein). A mean value is introduced to get the error in this study, by which an unexpected releasing event (we call it mal-releasing) due to some unknown abrupt disturbance can be avoided. Moreover, the data-releasing rate can be further reduced, which will be verified in Section 4.*

Remark 3. *ETMs in some existing literature mainly focus on reducing NDR for the system with suitable control performance. To get better control performance, the item in (4) with δ_2 is introduced, which plays a crucial role in enhancing the data-releasing rate during the period when the system is disturbed by external signals while it keeps a low level in the other period.*

The objective of this study is to design a dynamic output feedback controller with the form of (3) such that the system with stochastic nonlinearity in (1) satisfies

- The instants of data-releasing are decided by (5).
- For the given disturbance attenuation $\gamma > 0$, the system (1) is mean square stable and under zero initial state condition it satisfies $\mathbb{E}\left\{\int_{t_0}^{\infty} z^T(s)z(s)ds\right\} \leq \mathbb{E}\left\{\int_{t_0}^{\infty} \gamma^2\omega^T(s)\omega(s)ds\right\}$.

3. Co-design of the controller and ETM

In this section, we are in a position to design the controller in (3) for the system (1) under the proposed new ETM. Firstly, the closed-loop control system is modeled as a time-delay system with consideration of the ETM proposed in subsection 2.3.

Define $a_k^l = t_k h + lh + d_{t_k}^l$ and $\mathcal{L}_k^l \triangleq [a_k^l, a_k^{l+1})$. Let $\mathcal{L}_k = \cup_{l=0}^{l_M}\mathcal{L}_k^l$, and one can know that $d_{t_k}^0 = d_{t_k}$ and $d_{t_k}^{l_M+1} = d_{t_{k+1}}$, which are networked induced delays at instants $t_k h$ and $t_{k+1}h$, respectively. $d_{t_k}^l$ for $l \in \mathscr{L}_k \backslash \{0\}$ is an artificial delay to guarantee the interval $\mathcal{L}_k^l$ is meaningful.

Define

$$d(t) = t - t_k h \tag{6}$$

for $t \in \mathcal{L}_k^l$. Then it follows that

$$\underline{d} \leq d_{t_k} \leq d(t) \leq h + \overline{d} = d_M \tag{7}$$

where $\underline{d} \triangleq \min\{d_{t_k}\}$, $\overline{d} \triangleq \max\{d_{t_k}\}$. Recalling the definition of $e(t)$ and $d(t)$, we have

$$y(t_k h) = 2e(t) + y(t_k h + lh) = 2e(t) + y(t - d(t)) \tag{8}$$

for $t \in \mathcal{L}_k^l$. Combining (1) , (3) and (8) , we can obtain the following dynamic equation with a compact form for $t \in \mathcal{L}_k^l$

$$\dot{\tilde{x}}(t) = \mathcal{A}\tilde{x}(t) + \mathcal{B}_1\tilde{x}(t - d(t)) + \mathcal{B}_2 e(k) + \mathcal{B}_3\omega(t) + \alpha(t)\mathcal{B}_4 f_1(t, x(t)) + (1 - \alpha(t))\mathcal{B}_4 f_2(t, x(t)) \tag{9}$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}, \mathcal{A} = \begin{bmatrix} A & B_1 K \\ 0 & A_{c1} \end{bmatrix}, \mathcal{B}_1 = \begin{bmatrix} 0 & 0 \\ B_c C_1 & A_{c2} \end{bmatrix},$$
$$\mathcal{B}_2 = \begin{bmatrix} 0 \\ 2B_c \end{bmatrix}, \mathcal{B}_3 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}, \mathcal{B}_4 = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Define $\Gamma = \begin{bmatrix} \mathcal{A} & 0 & \mathcal{B}_1 & 0 & \mathcal{B}_2 & \mathcal{B}_3 & \alpha\mathcal{B}_4 & (1-\alpha)\mathcal{B}_4 \end{bmatrix}$, and $\xi(t) = col\{\tilde{x}(t), \tilde{x}(t - \underline{d}), \tilde{x}(t - d(t)), \tilde{x}(t - d_M), e(t), \omega^T(t), f_1(t, x(t)), f_2(t, x(t))\}$, the dynamic (9) can be re-written as

$$\dot{\tilde{x}}(t) = \Gamma\xi(t) + (\alpha(t) - \alpha)\left(\mathcal{B}_4 f_1(t, x(t)) - \mathcal{B}_4 f_2(t, x(t))\right) \tag{10}$$

The ETM condition proposed in Section 2.3 is equivalent to

$$\left(1 + \frac{\delta_2^2}{4\delta_1}\right) e(t)^T \Omega e(t) \leq \left[\sqrt{\delta_1} y(t_k h) + \frac{\delta_2}{2\sqrt{\delta_1}} e(t)\right]^T \Omega \left[\sqrt{\delta_1} y(t_k h) + \frac{\delta_2}{2\sqrt{\delta_1}} e(t)\right] \tag{11}$$

which leads to

$$\begin{aligned} \sigma_1 e(t)^T \Omega e(t) &\leq [\sigma_2 y(t - d(t)) + \sigma_3 e(t)]^T \Omega [\sigma_2 y(t - d(t)) + \sigma_3 e(t)] \\ &= [\sigma_2 C_1 E\tilde{x}(t - d(t)) + \sigma_3 e(t)]^T \Omega [\sigma_2 C_1 E\tilde{x}(t - d(t)) + \sigma_3 e(t)] \end{aligned} \tag{12}$$

for $t \in \mathcal{L}_k^l$, where $\sigma_1 = 1 + \frac{\delta_1^2}{4\delta_1}, \sigma_2 = \sqrt{\delta_1}, \sigma_3 = \frac{4\delta_1 + \delta_2}{2\sqrt{\delta_1}}, E = [I \quad 0]$.

The following lemmas are useful in deriving the main results.

Lemma 1. *[13] For any constant matrix $R_1 > 0 \in \mathbb{R}^{n\times n}$, scalars $0 \le d(t) \le \underline{d}$, and vector function $\dot{x} : [-\underline{d}, 0] \to \mathbb{R}^n$ such that the following integration is well defined, it holds that*

$$-\underline{d}\int_{t-\underline{d}}^{t} \dot{\tilde{x}}^T(s)R_1\dot{\tilde{x}}(s)ds \le \begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-\underline{d})\end{bmatrix}^T \begin{bmatrix}-R_1 & R_1\\ R_1 & -R_1\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-\underline{d})\end{bmatrix} \tag{13}$$

Lemma 2. *[32] Suppose M, N, and Ω are constant matrices of appropriate dimensions. Then*

$$(\tau(t) - \bar{\tau}_1)M + (\bar{\tau}_2 - \tau(t))N + \Omega < 0 \tag{14}$$

is true for any $\tau(t) \in [\bar{\tau}_1, \bar{\tau}_2]$, if and only if

$$(\bar{\tau}_2 - \bar{\tau}_1)M + \Omega < 0 \tag{15}$$

$$(\bar{\tau}_2 - \bar{\tau}_1)N + \Omega < 0 \tag{16}$$

Theorem 1. *For some given positive constants $\gamma, \underline{d}, d_M, \sigma_j$ $(j = 1, 2, 3)$, the system (10) is mean square stable with an H_∞ performance index γ, if there exist matrices $P > 0, \Omega > 0, Q_i > 0, R_i > 0$ and matrices N_i, M_i $(i = 1, 2)$ with appropriate dimensions such that*

$$\Pi_1^{(i)} = \begin{bmatrix}\Pi_{11} & * & *\\ \Pi_{21} & \Pi_{22} & *\\ \Pi_{31}^{(i)} & 0 & -R_2\end{bmatrix} < 0, (i = 1, 2) \tag{17}$$

where

$$\Pi_{11} = \begin{bmatrix}\Xi_1 & * & * & * & * & * & * & *\\ -R_1 & \Xi_2 & * & * & * & * & * & *\\ \mathcal{B}_1^T P & \Xi_3 & \Xi_4 & * & * & * & * & *\\ 0 & 0 & \Xi_5 & \Xi_6 & * & * & * & *\\ \mathcal{B}_2^T P & 0 & 0 & 0 & -\sigma_1\Omega & * & * & *\\ \mathcal{B}_3^T P & 0 & 0 & 0 & 0 & -\gamma^2 I & * & *\\ \alpha\mathcal{B}_4^T P & 0 & 0 & 0 & 0 & 0 & -\alpha I & *\\ (1-\alpha)\mathcal{B}_4^T P & 0 & 0 & 0 & 0 & 0 & 0 & -(1-\alpha)I\end{bmatrix},$$

$$\Pi_{21} = \begin{bmatrix}0 & 0 & \sigma_2 C_1 E & 0 & \sigma_3 I & 0 & 0 & 0\\ C_2 E & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \mathcal{A} & 0 & \underline{d}\mathcal{B}_1 & 0 & \underline{d}\mathcal{B}_2 & \underline{d}\mathcal{B}_3 & \alpha\underline{d}\mathcal{B}_4 & \Xi_7\\ \beta\mathcal{A} & 0 & \beta\mathcal{B}_1 & 0 & \beta\mathcal{B}_2 & \beta\mathcal{B}_3 & \alpha\beta\mathcal{B}_4 & \Xi_8\\ \Xi_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \Xi_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \Xi_{11} & \Xi_{12}\\ 0 & 0 & 0 & 0 & 0 & 0 & \Xi_{13} & \Xi_{14}\end{bmatrix},$$

$$\Pi_{31}^{(1)} = \begin{bmatrix}0 & \beta N_1^T & \beta N_2^T & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Pi_{31}^{(2)} = \begin{bmatrix}0 & 0 & \beta M_1^T & \beta M_2^T & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Pi_{22} = diag\{-\Omega^{-1}, -I, -R_1^{-1}, -R_2^{-1}, -I, -I, -R_1^{-1}, -R_2^{-1}, R_2\},$$

$$\Xi_1 = \mathcal{A}^T P + P\mathcal{A} + Q_1 + Q_2 - R_1, \Xi_2 = N_1 + N_1^T - Q_1 - R_1, \Xi_3 = N_2 - N_1^T,$$

$$\Xi_4 = -N_2 - N_2^T + M_1 + M_1^T, \Xi_5 = M_2 - M_1^T, \Xi_6 = -M_2 - M_2^T - Q_2,$$

$$\Xi_7 = (1-\alpha)\underline{d}\mathcal{B}_4, \Xi_8 = (1-\alpha)\beta\mathcal{B}_4, \Xi_9 = \sqrt{\alpha}S_1 E, \Xi_{10} = \sqrt{1-\alpha}S_2 E,$$

$$\Xi_{11} = \sqrt{\alpha(1-\alpha)}\underline{d}\mathcal{B}_4, \Xi_{12} = -\sqrt{\alpha(1-\alpha)}\underline{d}\mathcal{B}_4, \Xi_{13} = \sqrt{\alpha(1-\alpha)}\beta\mathcal{B}_4,$$

$$\Xi_{14} = -\sqrt{\alpha(1-\alpha)}\beta\mathcal{B}_4, \beta = \sqrt{d_M - \underline{d}}$$

PROOF. Choose the following Lyapunov-Krasovskii candidate for (10) [36]

$$V(t) = V_1(t) + V_2(t) + V_2(t) \tag{18}$$

where

$$\begin{aligned}
V_1(t) &= \tilde{x}^T(t)P\tilde{x}(t)\\
V_2(t) &= \int_{t-\underline{d}}^{t} \tilde{x}^T(s)Q_1\tilde{x}(s)ds + \int_{t-d_M}^{t} \tilde{x}^T(s)Q_2\tilde{x}(s)ds\\
V_3(t) &= \underline{d}\int_{t-\underline{d}}^{t}\int_{s}^{t} \dot{\tilde{x}}^T(v)R_1\dot{\tilde{x}}(v)dvds + \int_{t-d_M}^{t-\underline{d}}\int_{s}^{t} \dot{\tilde{x}}^T(v)R_2\dot{\tilde{x}}(v)dvds
\end{aligned}$$

For $t \in \mathcal{L}_k^l$, taking derivation on $V_i(t)(i = 1, 2, 3)$ and taking the expectation on them yields that

$$\begin{aligned}
&\mathbb{E}\left\{\dot{V}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\right\}\\
&= \mathbb{E}\left\{2\dot{\tilde{x}}^T(t)P\tilde{x}(t) + \alpha f_1^T(t, x(t))f_1(t, x(t))\right.\\
&-\alpha f_1^T(t, x(t))f_1(t, x(t)) + (1-\alpha)f_2^T(t, x(t))f_2(t, x(t))\\
&\left.-(1-\alpha)f_2^T(t, x(t))f_2(t, x(t)) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\right\}\\
&+ \mathbb{E}\left\{\tilde{x}^T(t)Q_1\tilde{x}(t) - \tilde{x}(t-\underline{d})Q_1\tilde{x}(t-\underline{d})\right\}\\
&+ \mathbb{E}\left\{\tilde{x}^T(t)Q_2\tilde{x}(t) - \tilde{x}(t-d_M)Q_2\tilde{x}(t-d_M)\right\}\\
&+ \mathbb{E}\left\{d_1^2\dot{\tilde{x}}^T(t)R_1\dot{\tilde{x}}(t) + (d_M-\underline{d})\dot{\tilde{x}}^T(t)R_2\dot{\tilde{x}}(t)\right\}\\
&- \mathbb{E}\left\{\underline{d}\int_{t-\underline{d}}^{t} \dot{\tilde{x}}^T(s)R_1\dot{\tilde{x}}(s)ds - \int_{t-d_M}^{t-\underline{d}} \dot{\tilde{x}}^T(s)R_2\dot{\tilde{x}}(s)ds\right\}\\
&+ \mathbb{E}\left\{2\xi^T(t)N\left[\tilde{x}(t-\underline{d}) - \tilde{x}(t-d(t)) - \int_{t-d(t)}^{t-\underline{d}} \dot{\tilde{x}}(s)ds\right]\right\}\\
&+ \mathbb{E}\left\{2\xi^T(t)M\left[\tilde{x}(t-d(t)) - \tilde{x}(t-d_M)) - \int_{t-d_M}^{t-d(t)} \dot{\tilde{x}}(s)ds\right]\right\}
\end{aligned}$$

where $N = \begin{bmatrix}0 & N_1^T & N_2^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$, and $M = \begin{bmatrix}0 & 0 & M_1^T & M_2^T & 0 & 0 & 0 & 0\end{bmatrix}^T$.

Notice that $\mathbb{E}\{\alpha(t) - \alpha\} = 0$. Then we have

$$\mathbb{E}\left\{2\dot{\tilde{x}}^T(t)P\tilde{x}(t)\right\} \leq \mathbb{E}\left\{2\xi^T(t)\Gamma^T Px(t)\right\} \tag{19}$$

From the definition of $f_1(t, x(t))$ and $f_2(t, x(t))$ in (2), one can know that

$$\begin{aligned}
\alpha\mathbb{E}\left\{f_1^T(t, x(t))f_1(t, x(t))\right\} &\leq \alpha\mathbb{E}\left\{\tilde{x}^T(t)E^TS_1^TS_1E\tilde{x}(t)\right\}\\
(1-\alpha)\mathbb{E}\left\{f_2^T(t, x(t))f_2(t, x(t))\right\} &\leq (1-\alpha)\mathbb{E}\left\{\tilde{x}^T(t)E^TS_2^TS_2E\tilde{x}(t)\right\}
\end{aligned} \tag{20}$$

Then

$$\begin{aligned}
&\mathbb{E}\left\{\dot{\tilde{x}}^T(t)R_i\dot{\tilde{x}}(t)\right\} \leq \mathbb{E}\left\{\xi^T(t)\Gamma^TR_i\Gamma\xi(t)\right\} + \mathbb{E}\left\{(1-\alpha)\tilde{x}^T(t)E^TS_2^TS_2E\tilde{x}(t)\right\}\\
&+ \alpha(1-\alpha)\mathbb{E}\left\{(\mathcal{B}_4f_1(t, x(t)) - \mathcal{B}_4f_2(t, x(t)))^T R_i(\mathcal{B}_4f_1(t, x(t)) - \mathcal{B}_4f_2(t, x(t)))\right\}\\
&- \mathbb{E}\left\{\alpha f_1^T(t, x(t))f_1(t, x(t)) - (1-\alpha)\tilde{x}^T(t)E^TS_2^TS_2E\tilde{x}(t) + \alpha\tilde{x}^T(t)E^TS_1^TS_1E\tilde{x}(t)\right\}
\end{aligned}$$

For matrices M, N and $R_2 > 0$, it follows that [39]

$$-2\xi^T(t)N\int_{t-d(t)}^{t-\underline{d}} \dot{\tilde{x}}(s)ds \leq (d(t)-\underline{d})\xi^T(t)NR_2^{-1}N^T\xi(t) + \int_{t-d(t)}^{t-\underline{d}} \dot{\tilde{x}}^T(s)R_2\dot{\tilde{x}}(s)ds \tag{21}$$

$$-2\xi^T(t)M\int_{t-d_M}^{t-d(t)}\dot{x}(s)ds \leq (d_M - d(t))\xi^T(t)MR_2^{-1}M^T\xi(t) + \int_{t-d_M}^{t-d(t)}\dot{x}^T(s)R_2\dot{x}(s)ds \tag{22}$$

Combing the equivalent event-triggering condition in (12) and using Lemma 1, we have

$$\begin{aligned}&\mathbb{E}\left\{\dot{V}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\right\}\\ &\leq \mathbb{E}\left\{\xi^T(t)\left(\Pi_{11} - \Pi_{21}^T\Pi_{22}^{-1}\Pi_{21} + (d(t)-\underline{d})NR_2^{-1}N^T + (d_M - d(t))MR_2^{-1}M^T\right)\xi(t)\right\}\end{aligned} \tag{23}$$

By the Schur complement, one obtains

$$\Pi_{11} - \Pi_{21}^T\Pi_{22}^{-1}\Pi_{21} + (d_M - \underline{d})NR_2^{-1}N^T \leq 0 \tag{24}$$

$$\Pi_{11} - \Pi_{21}^T\Pi_{22}^{-1}\Pi_{21} + (d_M - \underline{d})MR_2^{-1}M^T \leq 0 \tag{25}$$

is equivalent to Eq. (17). From Lemma 2, one can see that (17) is a sufficient condition to guarantee

$$\Pi_{11} - \Pi_{21}^T\Pi_{22}^{-1}\Pi_{21} + (d(t)-\underline{d})NR_2^{-1}N^T + (d_M - d(t))MR_2^{-1}M^T \leq 0 \tag{26}$$

which implies that

$$\mathbb{E}\left\{\dot{V}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\right\} < 0 \tag{27}$$

Under zero initial condition, integrating both sides of Eq. (27) from 0 to t and let $t \to \infty$, we have

$$\mathbb{E}\left\{\int_{t_0}^{\infty} z^T(s)z(s)ds\right\} \leq \mathbb{E}\left\{\int_{t_0}^{\infty}\gamma^2\omega^T(s)\omega(s)ds\right\} \tag{28}$$

With the condition of $\omega(t) = 0$, we can conclude that $\mathbb{E}\{\dot{V}(t)\} < 0$ from Eq. (27). The proof can be completed.

Theorem 1 gives a sufficient condition to guarantee the system (1) to be mean square stable with an H_∞ performance index γ under the proposed ETM. Theorem 1 is obtained using Jensen inequality (i.e. Lemma 1). However, one can use some free-weighting matrix-based integral inequalities (e.g. [29]) to derive some better results by following the similar procedure as above. Next, we aim to design the dynamic output feedback controller gain A_{c1}, A_{c2}, B_c and K in (3) based on Theorem 1 to ensure that the system (1) is mean square stable with an H_∞ norm bound γ.

Theorem 2. *For some given positive constants $\gamma, \underline{d}, d_M, \sigma_i$ $(i = 1,2,3), \rho, \rho_j$ $(j = 1,2)$, the system (10) is mean square stable with an H_∞ performance index γ, if there exist matrices $X > 0, Y > 0, \Omega > 0, \widetilde{Q}_i > 0, \widetilde{R}_i > 0$ and $\widetilde{N}_i, \widetilde{M}_i$ and matrices $W_j(i = 1,2 \;\; j = 1,...,4)$ with appropriate dimensions such that*

$$\widetilde{\Pi}_1^{(i)} = \begin{bmatrix}\widetilde{\Pi}_{11} & * & * \\ \widetilde{\Pi}_{21} & \widetilde{\Pi}_{22} & * \\ \widetilde{\Pi}_{31}^{(i)} & 0 & -\widetilde{R}_2\end{bmatrix} < 0, (i = 1,2), \tag{29}$$

$$Z = \begin{bmatrix} X & * \\ I & Y\end{bmatrix} > 0 \tag{30}$$

where

$$\widetilde{\Pi}_{11} = \begin{bmatrix}\widetilde{\Xi}_1 & * & * & * & * & * & * & * \\ -\widetilde{R}_1 & \widetilde{\Xi}_2 & * & * & * & * & * & * \\ \Psi_1^T & \widetilde{\Xi}_3 & \widetilde{\Xi}_4 & * & * & * & * & * \\ 0 & 0 & \widetilde{\Xi}_5 & \widetilde{\Xi}_6 & * & * & * & * \\ \Psi_2^T & 0 & 0 & 0 & -\sigma_1\Omega & * & * & * \\ \Psi_3^T & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * \\ \alpha\Psi_4^T & 0 & 0 & 0 & 0 & 0 & -\alpha I & * \\ (1-\alpha)\Psi_4^T & 0 & 0 & 0 & 0 & 0 & 0 & -(1-\alpha)I\end{bmatrix},$$

$$\widetilde{\Pi}_{21} = \begin{bmatrix} 0 & 0 & \sigma_2\widetilde{\Xi}_7 & 0 & \sigma_3 I & 0 & 0 & 0 \\ \widetilde{\Xi}_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \underline{d}\Psi_0 & 0 & \underline{d}\Psi_1 & 0 & \underline{d}\Psi_2 & \underline{d}\Psi_3 & \alpha\underline{d}\Psi_4 & \widetilde{\Xi}_9 \\ \beta\Psi_0 & 0 & \beta\Psi_1 & 0 & \beta\Psi_2 & \beta\Psi_3 & \alpha\beta\Psi_4 & \widetilde{\Xi}_{10} \\ \widetilde{\Xi}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \widetilde{\Xi}_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{\Xi}_{13} & \widetilde{\Xi}_{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{\Xi}_{15} & \widetilde{\Xi}_{16} \end{bmatrix},$$

$$\Psi_0 = \begin{bmatrix} AX + BW_1 & A \\ W_4 & YA \end{bmatrix}, \Psi_1 = \begin{bmatrix} 0 & 0 \\ W_3 & W_2C_1 \end{bmatrix}, \Psi_2 = \begin{bmatrix} 0 \\ 2W_2 \end{bmatrix}, \Psi_3 = \begin{bmatrix} B_2 \\ YB_2 \end{bmatrix}, \Psi_4 = \begin{bmatrix} I \\ Y \end{bmatrix},$$

$$\widetilde{\Pi}_{31}^{(1)} = \begin{bmatrix} 0 & \beta\widetilde{N}_1^T & \beta\widetilde{N}_2^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Pi}_{31}^{(2)} = \begin{bmatrix} 0 & 0 & \beta\widetilde{M}_1^T & \beta\widetilde{M}_2^T & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Pi}_{22} = diag\left\{-\rho I + \rho^2\Omega, -I, -\rho_1 Z + \rho_1^2\widetilde{R}_1, -\rho_2 Z + \rho_2^2\widetilde{R}_2, -I, -I, -\rho_1 Z + \rho_1^2\widetilde{R}_1, -\rho_2 Z + \rho_2^2\widetilde{R}_2, \widetilde{R}_2\right\},$$

$$\widetilde{\Xi}_1 = \Psi_0^T + \Psi_0 + \widetilde{Q}_1 + \widetilde{Q}_2 - \widetilde{R}_1, \widetilde{\Xi}_2 = \widetilde{N}_1 + \widetilde{N}_1^T - \widetilde{Q}_1 - \widetilde{R}_1, \widetilde{\Xi}_3 = \widetilde{N}_2 - \widetilde{N}_1^T,$$

$$\widetilde{\Xi}_4 = -\widetilde{N}_2 - \widetilde{N}_2^T + \widetilde{M}_1 + \widetilde{M}_1^T, \widetilde{\Xi}_5 = \widetilde{M}_2 - \widetilde{M}_1^T, \widetilde{\Xi}_6 = -\widetilde{M}_2 - \widetilde{M}_2^T - \widetilde{Q}_2,$$

$$\widetilde{\Xi}_7 = [C_1X \ \ C_1], \widetilde{\Xi}_8 = [C_2X \ \ C_2], \widetilde{\Xi}_9 = (1-\alpha)\underline{d}\Psi_4, \widetilde{\Xi}_{10} = (1-\alpha)\beta\Psi_4,$$

$$\widetilde{\Xi}_{11} = \left[\sqrt{\alpha}S_1X \ \ \sqrt{\alpha}S_1\right], \widetilde{\Xi}_{12} = \left[\sqrt{1-\alpha}S_2X \ \ \sqrt{1-\alpha}S_2\right],$$

$$\widetilde{\Xi}_{13} = \sqrt{\alpha(1-\alpha)}\underline{d}\Psi_4, \widetilde{\Xi}_{14} = -\sqrt{\alpha(1-\alpha)}\Psi_4, \widetilde{\Xi}_{15} = \sqrt{\alpha(1-\alpha)}\beta\Psi_4$$

$$\widetilde{\Xi}_{16} = -\sqrt{\alpha(1-\alpha)}\beta\Psi_4, \beta = \sqrt{d_M - \underline{d}}$$

Proof. Introducing a non-singular matrix $U \in \mathbb{R}^{n\times n}$, such that $P = \begin{bmatrix} Y & U^T \\ U & Y_1 \end{bmatrix} > 0$, where $Y_1 = U(Y - X^{-1})U^T$, one can know that $Y_1 > 0$ from Eq. (30) by using Schur complement, and $Y > 0$ as well.

Notice that $(\rho_i\widetilde{R}_i - Z)^T\widetilde{R}_i^{-1}(\rho_i\widetilde{R}_i - Z) > 0$ for $\widetilde{R}_i > 0, Z > 0$ and $\rho_i > 0$. Then

$$-Z\widetilde{R}_i^{-1}Z \leq -\rho_i Z + \rho_i^2\widetilde{R}_i, (i = 1, 2) \tag{31}$$

Similarly, the following is true

$$-\Omega^{-1} \leq -\rho I + \rho^2\Omega \tag{32}$$

In order to get an LMI criterion, we denote

$$\begin{cases} W_1 = KU^{-T}(I - YX) \\ W_2 = U^T B_c \\ W_3 = U^T B_c C_1 X + U^T A_{c_2} U^{-T}(I - YX) \\ W_4 = YAX + YB_c K U^{-T}(I - YX) + U^T A_{c_1} U^{-T}(I - YX) \end{cases} \tag{33}$$

Define

$$\begin{aligned} J_1 &= \begin{bmatrix} X & I \\ U^{-T}(I - YX) & 0 \end{bmatrix}, J_2 = \begin{bmatrix} I & Y \\ 0 & U \end{bmatrix}, \\ \Phi_1 &= diag\{J_1, J_1, J_1, J_1, I, I, I, I\}, \\ \Phi_2 &= diag\{I, I, J_2, J_2, I, I, J_2, J_2\} \end{aligned}$$

Let $\Phi = diag\{\Phi_1, \Phi_2, J_1\}$ and define $\widetilde{R}_i = J_1^T R_i J_1, \widetilde{Q}_i = J_1^T Q_i J_1$ for $i = 1, 2$. Pre- and post-multiplying (17) with Φ and its transpose, and combing Eq. (31) - Eq. (33), we can conclude that (17) is a sufficient condition to guarantee (29) holds. The proof is thus completed.

From Eq. (33), we can obtain

$$\begin{cases} A_{c1} = U^{-T}(W_4 - YAX - YB_1W_1)(I - YX)^{-1}U^T \\ A_{c2} = U^{-T}(W_3 - W_2C_1X)(I - YX)^{-1}U^T \\ B_c = U^{-T}W_2 \\ K = W_1(I - YX)^{-1}U^T \end{cases} \tag{34}$$

Remark 4. *A less conservative result can be achieved if one chooses CCL algorithm [12]. To decrease the computational complexity, the inequalities (31) and (32) are used to deal with the nonlinear iterms $-\Omega^{-1}$ and $-\widetilde{R}_i^{-1}$, respectively, in this study.*

Remark 5. *Some new methods, such as the method in [14, 41] may bring less conservative results, however, we mainly focus on developing a novel ETM for the NCSs in this study. Lemma 2 is used for deriving convenience.*

The error $e(t)$ in this study is defined as $e(t) = y(t_kh) - \bar{y}(t_kh)$. If choosing the error $e(t) = y(t_kh) - y(t_kh + lh)$ for the system without the item of nonlinearity, then we have the following corollary.

Corollary 1. *For some given positive constants $\gamma, \underline{d}, d_M, \sigma_i$ $(i = 1,2,3), \rho, \rho_j$ $(j = 1,2)$, the system (10) is mean square stable with an H_∞ performance index γ, if there exist matrices $X > 0, Y > 0, \Omega > 0, \widetilde{Q}_i > 0, \widetilde{R}_i > 0$ and $\widetilde{N}_i, \widetilde{M}_i$ and matrices $W_j (i = 1,2 \;\; j = 1,...,4)$ with appropriate dimensions such that*

$$\hat{\Pi}_1^{(i)} = \begin{bmatrix} \hat{\Pi}_{11} & * & * \\ \hat{\Pi}_{21} & \hat{\Pi}_{22} & * \\ \hat{\Pi}_{31}^{(i)} & 0 & -\widetilde{R}_2 \end{bmatrix} < 0, (i = 1,2) \tag{35}$$

$$Z = \begin{bmatrix} X & * \\ I & Y \end{bmatrix} > 0 \tag{36}$$

where

$$\hat{\Pi}_{11} = \begin{bmatrix} \widetilde{\Xi}_1 & * & * & * & * & * \\ -\widetilde{R}_1 & \widetilde{\Xi}_2 & * & * & * & * \\ \Psi_1^T & \widetilde{\Xi}_3 & \widetilde{\Xi}_4 & * & * & * \\ 0 & 0 & \widetilde{\Xi}_5 & \widetilde{\Xi}_6 & * & * \\ \hat{\Psi}_2^T & 0 & 0 & 0 & -\sigma_1\Omega & * \\ \Psi_3^T & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\hat{\Pi}_{21} = \begin{bmatrix} 0 & 0 & \sigma_2\widetilde{\Xi}_7 & 0 & \hat{\sigma}_3 I & 0 \\ \widetilde{\Xi}_8 & 0 & 0 & 0 & 0 & 0 \\ \underline{d}\Psi_0 & 0 & \underline{d}\Psi_1 & 0 & \underline{d}\hat{\Psi}_2 & \underline{d}\Psi_3 \\ \beta\Psi_0 & 0 & \beta\Psi_1 & 0 & \beta\hat{\Psi}_2 & \beta\Psi_3 \end{bmatrix},$$

$$\hat{\Psi}_2 = \begin{bmatrix} 0 \\ W_2 \end{bmatrix}, \hat{\sigma}_3 = \frac{2\delta_1 + \delta_2}{2\sqrt{\delta_1}}$$

$$\widetilde{\Pi}_{31}^{(1)} = \begin{bmatrix} 0 & \beta\widetilde{N}_1^T & \beta\widetilde{N}_2^T & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Pi}_{31}^{(2)} = \begin{bmatrix} 0 & 0 & \beta\widetilde{M}_1^T & \beta\widetilde{M}_2^T & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Pi}_{22} = diag\left\{-\rho I + \rho^2\Omega, -I, -\rho_1 Z + \rho_1^2\widetilde{R}_1, -\rho_2 Z + \rho_2^2\widetilde{R}_2, \widetilde{R}_2\right\}$$

the other parameters are the same as those in Theorem 2.

The parameters of dynamic output feedback controller A_{c1}, A_{c2}, B_c and K can be obtained from Eq. (34). However, Theorem 2 and Corollary 1 are independent on the non-singular matrix U, which means that it is hard to design the controller gains from (34). For this reason, we perform an irreducible transformation as $x_c(t) = U^{-T}\overline{x}_c(t)$ on the dynamic (3), yielding

$$\begin{cases} \dot{\overline{x}}_c(t) = \mathbb{A}_{c1}\overline{x}_c + \mathbb{A}_{c2}\overline{x}_c(t-d(t)) + \mathbb{B}_c y(t_k h) \\ u(t) = \mathbb{K}\overline{x}_c(t) \end{cases} \tag{37}$$

where $\mathbb{A}_{c1} = U^T A_{c1} U^{-T}, \mathbb{A}_{c2} = U^T A_{c2} U^{-T}, \mathbb{B}_c = U^T B_c, \mathbb{K} = KU^{-T}$. From (34), we have

$$\begin{cases} \mathbb{A}_{c1} = (W_4 - YAX - YB_1W_1)(I-YX)^{-1} \\ \mathbb{A}_{c2} = (W_3 - W_2C_1X)(I-YX)^{-1} \\ \mathbb{B}_c = W_2 \\ \mathbb{K} = W_1(I-YX)^{-1} \end{cases} \tag{38}$$

According to the analysis above, the dynamic output feedback control and ETM for the system with stochastic nonlinearity can be co-designed by the following algorithm:

Algorithm 1:

- Step 1: Choose the expectation of each nonlinearity α_i;
- Step 2: Get X, Y, W_i $(i = 1, 2, 3, 4)$, and Ω in (4) by solving LMIs (29) and (30) or (35) and (36);
- Step 3: The controller gain in (3) is derived from $(A_{c1}, A_{c2}, B_c, K) = (\mathbb{A}_{c1}, \mathbb{A}_{c2}, \mathbb{B}_c, \mathbb{K})$;
- Step 4: Determine the next releasing instant based on (5).

4. EXAMPLES

Example 1: Consider the inverted pendulum with the dynamics described by (1). The parameters are given by [16]

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -10 & -10 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, C_1^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, C_2^T = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{39}$$

Assume the control signal is transmitted over the network. The index of network in (7) is defined by $\overline{d} = 0.008s$, $\underline{d} = 0.004s$ and the sampling period is $h = 0.01s$. The initial condition, in this simulation, is given by $x(t) = [0.2 \quad -0.3 \quad 0.2]$.

Four cases listed in Table 1 are set up in this example. In Case I-III, the nonlinearities of the system are removed to avoid a confusion in comparison, where Case I uses a traditional ETM in [17]; A mean value is introduced to get the error $e(t)$ of the traditional ETM in Case II; The proposed ETM in this study is Case IV; The ETM condition of Case III has a same form as the one in Case IV, while the error $e(t)$ is given by a traditional way.

Table 1: Different cases of ETM in (4) $(\delta_1 = 0.006)$

	$\delta_i (i = 1, 2)$	$e(t)$
Case I	$\delta_2 = 0$	$e(t) = y(t_k h) - y(t_k h + lh)$
Case II	$\delta_2 = 0$	$e(t) = y(t_k h) - \frac{1}{2}[y(t_k h) + y(t_k h + lh)]$
Case III	$\delta_2 = 0.1$	$e(t) = y(t_k h) - y(t_k h + lh)$
Case IV	$\delta_2 = 0.1$	$e(t) = y(t_k h) - \frac{1}{2}[y(t_k h) + y(t_k h + lh)]$

By Algorithm 1, we can get the controller gains and the parameters of ETM under Case I, Case II and Case III with $\rho_1 = \rho_2 = \rho = 0.1$ and $\gamma = 5$ as follows:

Under Case I:

$$A_{c1} = \begin{bmatrix} 0.3052 & 0.5360 & -1.7257 \\ -18.0561 & -6.4285 & 2.0333 \\ -16.7396 & -4.8989 & -3.4639 \end{bmatrix}, A_{c2} = \begin{bmatrix} -1.3007 & -0.3763 & 0.5345 \\ 0.3085 & 0.0902 & -0.1281 \\ 0.4459 & 0.1591 & -0.3210 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -6.4311 & 1.1924 \\ 1.4921 & -0.3098 \\ 1.8865 & -1.4634 \end{bmatrix}, K = \begin{bmatrix} 0.1758 \\ 0.0473 \\ -0.0542 \end{bmatrix}^T, \Omega = \begin{bmatrix} 7.8917 & -0.3119 \\ -0.3119 & 5.4862 \end{bmatrix} \tag{40}$$

Under Case II:

$$A_{c1} = \begin{bmatrix} -0.2342 & 0.2412 & -0.9603 \\ -33.7735 & -7.2517 & 2.0053 \\ -31.8139 & -5.7639 & -2.6311 \end{bmatrix}, A_{c2} = \begin{bmatrix} -1.0533 & -0.1885 & 0.2586 \\ 0.4989 & 0.0913 & -0.1204 \\ 0.5548 & 0.1233 & -0.2245 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -2.9989 & 0.3612 \\ 1.3150 & -0.2239 \\ 1.3655 & -0.8872 \end{bmatrix}, K = \begin{bmatrix} 0.3336 \\ 0.0569 \\ -0.0630 \end{bmatrix}^T, \Omega = \begin{bmatrix} 13.4947 & -0.4575 \\ -0.4575 & 8.7400 \end{bmatrix} \tag{41}$$

Under Case III:

$$A_{c1} = \begin{bmatrix} -0.5945 & 0.2312 & -0.9653 \\ -63.4559 & -21.3934 & 12.3388 \\ -65.8097 & -21.0398 & 9.4696 \end{bmatrix}, A_{c2} = \begin{bmatrix} -1.3508 & -0.4000 & 0.4413 \\ 0.2578 & 0.0710 & -0.0530 \\ 0.2026 & 0.0988 & -0.1631 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -5.9569 & 0.7028 \\ 0.8869 & 0.3076 \\ 0.2978 & -1.4977 \end{bmatrix}, K = \begin{bmatrix} 0.2718 \\ 0.0829 \\ -0.0727 \end{bmatrix}^T, \Omega = \begin{bmatrix} 8.4449 & 0.0210 \\ 0.0210 & 6.5718 \end{bmatrix} \tag{42}$$

In practice, the output signal may have a jitter in processing of measurement, which will lead to some unexpected releasing events. To reduce the probability of mal-releasing, a mean value is introduced to get the error $e(t)$. Furthermore, the number of data-releasing can be greatly reduced.

For clear description, we define

$$\text{date-releasing rate } (\eta) = \frac{\text{the number of data-releasing (NDR)}}{\text{the number of data-sampling (NDS)}} \tag{43}$$

Here we set the disturbance $\omega(t)$ occurs at every sampling instant during 10s-15s as follows

$$\omega_1(t) = \begin{cases} 0.02, & t = t_k h \text{ and } t \in [10, 15] \\ 0 & \text{others} \end{cases} \tag{44}$$

Under this pulsatile disturbance, the system states and triggering instants under Case I and Case II are shown in the Figure 1 and Figure 2, respectively. The data releasing rates are $\frac{314}{2501}$ and $\frac{162}{2501}$ for those two cases, respectively, while the control performance has no clear changes by comparing the state responses in Figure 1 and Figure 2. A lower data-releasing rate can be achieved by introducing a new definition of $e(t)$ in Case II. Thus the network burden can be mitigated.

To illustrate our proposed scheme by introducing the item with δ_2 ($\delta_2 \neq 0$) in (4) can enhance the data-releasing rate when the system disturbed by external disturbance, we reset the disturbance as

$$\omega_2(t) = \begin{cases} 0.01, & t \in [18, 22] \\ 0, & \text{others} \end{cases} \tag{45}$$

Figure 3 and Figure 4 show the responses of the system with $\omega_2(t)$ under Case I and Case III, respectively. Table 2 gives the statistics under those two cases. The data releasing rate in Case I and Case III are $\frac{434}{3501}$ and $\frac{328}{3501}$, respectively.

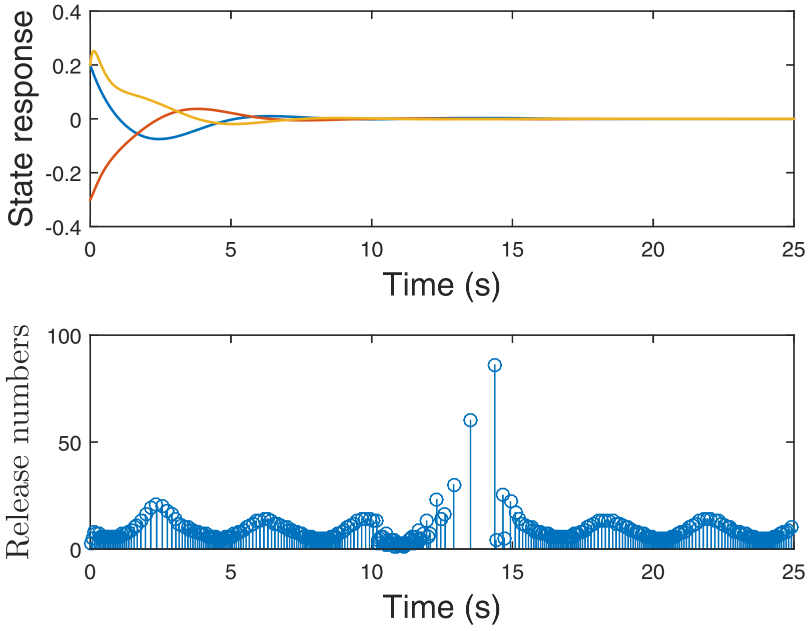

Figure 1: State responses and triggering instants under Case I and $\omega_1(t)$

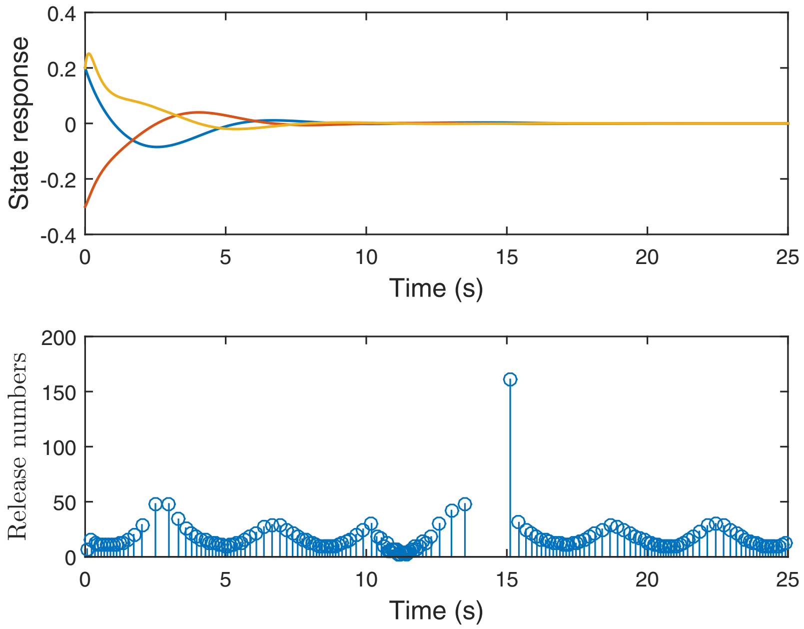

Figure 2: State responses and triggering instants under Case II

It demonstrates that the proposed ETM can reduce the data releasing rate to mitigate the burden of network effectively. Next, we further examine the NDR during 18s-22s when the disturbance occurs. It can be found that much more sampling data are released into the network by comparing the system under Case III and Case I during this period.

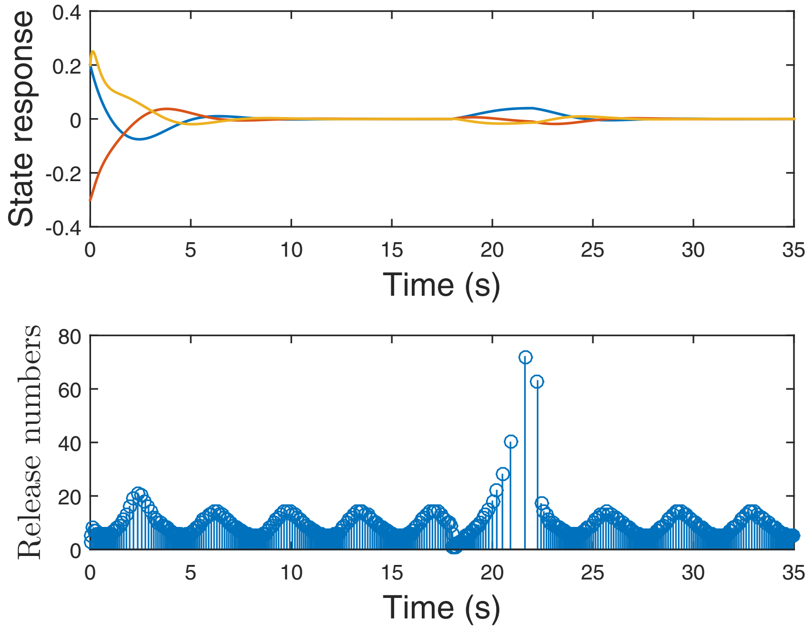

Figure 3: State responses and triggering instants under Case I and $\omega_2(t)$

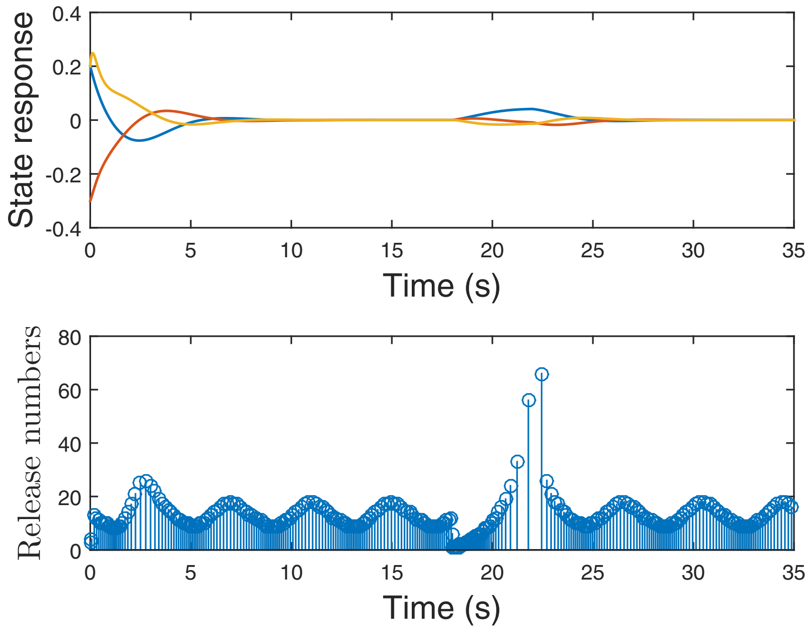

Figure 4: State responses and triggering instants under Case III and $\omega_2(t)$

That is, the controller can receive more information from the system when it is interfered with the external disturbance by using the proposed method.

Next, we study the performance for the system with stochastic nonlinearity under Case IV. The parameters of

Table 2: NDS and NDR of the system with $\omega_2(t)$

condition	NDS	NDR	NDR in [18s,22s]
CASE I	3501	434	54
CASE III	3501	328	81

stochastic nonlinearity are given by $F = 2, \alpha = 0.8, S_1 = diag[0.1\ 0.1\ 0.1], S_2 = diag[0.3\ 0.3\ 0.3]$. According to Algorithm 1, the parameters of dynamic controller and ETM can be obtained as

$$A_{c1} = \begin{bmatrix} -12.2791 & -4.9506 & 3.7549 \\ -124.6644 & -54.2904 & 41.1158 \\ -141.0967 & -60.2342 & 44.2077 \end{bmatrix}, A_{c2} = \begin{bmatrix} -2.1377 & -0.2793 & 0.5484 \\ 1.9096 & 0.9562 & -0.3971 \\ 1.0456 & 0.9029 & -0.8446 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -1.6383 & 0.1120 \\ -0.4790 & 0.5221 \\ -0.6069 & -0.2648 \end{bmatrix}, K = \begin{bmatrix} 7.8342 \\ 3.2055 \\ -2.8420 \end{bmatrix}^T, \Omega = \begin{bmatrix} 9.8990 & -0.1415 \\ -0.1415 & 7.2437 \end{bmatrix} \tag{46}$$

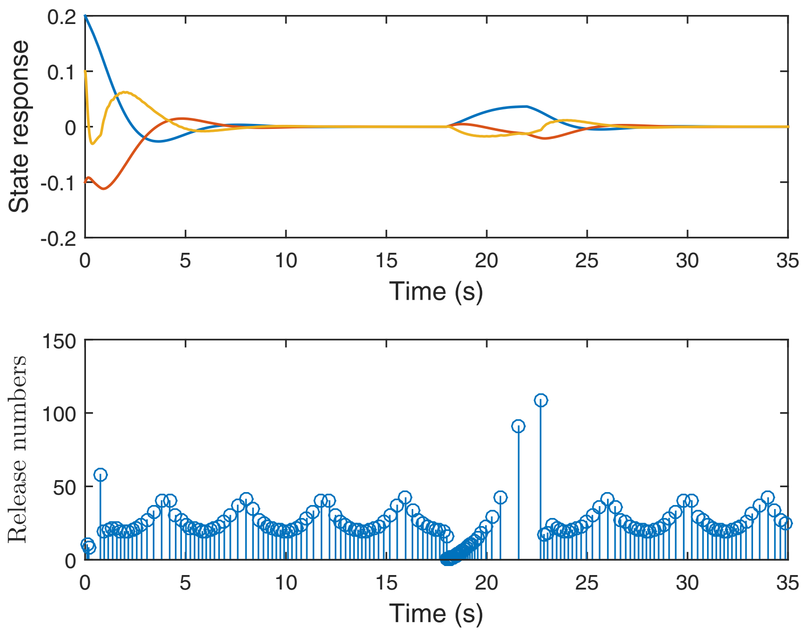

Figure 5: State responses and triggering instants under Case IV and $\omega_2(t)$

Figure 5 shows the state responses and event-triggering instants of the system by using the proposed ETM, from which one can see the system has a good control performance with a lower data releasing rate. Furthermore, the NDR during the period when the system is disturbed is much more than the other period. The controller can receive more data under this period such that the system to achieve a better control performance.

5. Conclusion

In this paper, an improved ETM for networked control systems with probabilistic nonliearities has been proposed. As a result, the data-releasing rate can be further reduced. It has been shown that this ETM is more helpful in improving the control performance with less data-releasing rate than some conventional ETMs, which contributes to the new event-triggering condition introduced in the ETM. Furthermore, a mean value has been introduced to the error such that the data-releasing number can be further reduced, since unexpected triggering event generated by some unknown noises is avoided effectively. An example with 4 cases has been given to demonstrate the effectiveness of the proposed method. It should be pointed out that some network-induced phenomena, such as packet dropouts and packet disorders [8] are not considered in this study, which motivates our future study.

6. Acknowledgement

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References

[1] J. Almeida, C. Silvestre, A. Pascoal, Self-triggered output feedback control of linear plants in the presence of unknown disturbances, IEEE Trans. Autom. Control 59 (11) (2014) 3040-3045.
[2] L. Ding, G. Guo, Distributed event-triggered H_∞ consensus filtering in sensor networks, Signal Process. 108 (2015) 365–375.
[3] L. Ding, Q.-L. Han, X. Ge, X.-M. Zhang, An overview of recent advances in event-triggered consensus of multi-agent systems, IEEE Trans. Cyber. 48(4) (2018) 1110–1123.
[4] L. Ding, Q.-L. Han, G. Guo, Network-based leader-following consensus for distributed multi-agent systems, Automatica 49 (7) (2013) 2281–2286.
[5] V. S. Dolk, D. P. Borgers, W. P. M. H. Heemels, Output-based and decentralized dynamic event-triggered control with guaranteed $\mathcal{L}_p$- gain performance and zeno-freeness, IEEE Trans. Autom. Control 62 (1) (2016) 34–49.
[6] M. C. F. Donkers, W. P. M. H. Heemels, Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved and decentralized event-triggering, IEEE Trans. Autom. Control 57 (6) (2012) 1362–1376.
[7] D. Du, B. Qi, M. Fei, Z. Wang, Quantized control of distributed event-triggered networked control systems with hybrid wired-wireless networks communication constraints, Inf. Sci. 380 (2017) 74–91.
[8] X. Ge, F. Yang, Q.-L. Han, Distributed networked control systems: A brief overview, Inf. Sci. 380 (2017) 117–131.
[9] X. Ge, Q.-L. Han, Distributed event-triggered H_∞ filtering over sensor networks with communication delays, Inf. Sci. 291 (2015) 128–142.
[10] X. Ge, Q.-L. Han, Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism, IEEE Trans. Ind. Electron. 64 (10) (2017) 8118–8127.
[11] X. Ge, Q.-L. Han, X.-M. Zhang, Achieving cluster formation of multi-agent systems under aperiodic sampling and communication delays, IEEE Trans. Ind. Electron. 65 (4) (2018) 3417-3426.
[12] L. E. Ghaoui, F. Oustry, M. Aitrami, A cone complementarity linearization algorithm for static output-feedback and related problems, IEEE Trans. Autom. Control 42 (8) (1997) 1171–1176.
[13] K. Gu , J. Chen , V.L. Kharitonov , Stability of Time-Delay Systems, Springer Science & Business Media, 2003 .
[14] Z. Gu, P. Shi, D. Yue, An adaptive event-triggering scheme for networked interconnected control system with stochastic uncertainty, Int. J. Robust Nonlin. Control 27 (2) (2017) 236–251.
[15] Z. Gu, D. Yue, E. Tian, On designing of an adaptive event-triggered communication scheme for nonlinear networked interconnected control systems, Inf. Sci. 422 (2018) 257–270.
[16] H. Han, Sliding-mode output feedback control design, IEEE Trans. Ind. Electron. 55 (11) (2008) 4047–4054.
[17] S. Hu, D. Yue, L_2-gain analysis of event-triggered networked control systems: a discontinuous lyapunov functional approach, Int. J. Robust Nonlin. Control 23 (11) (2013) 1277-1300.
[18] D. Liu, G.-H. Yang, Event-triggered non-fragile control for linear systems with actuator saturation and disturbances, Inf. Sci. 429 (2018) 1–11.
[19] M. Mazo, P. Tabuada, Decentralized event-triggered control over wireless sensor/actuator networks, IEEE Trans. Autom. Control 56 (10) (2011) 2456-2461.
[20] C. Peng, M. Wu, X. Xie, Y. Wang, Event-triggered Predictive Control for Networked Nonlinear Systems with Imperfect Premise Matching, IEEE Trans. Fuzzy Syst. (2018), DOI : 10.1109/TFUZZ.2018.2799187.
[21] C. Peng, J. Zhang, Event-triggered output-feedback H_∞ control for networked control systems with time-varying sampling, IET Control Theory Appl. 9 (9) (2015) 1384–1391.
[22] C. Peng, J. Zhang, H. Yan, Adaptive Event-Triggering H_∞ Load Frequency Control for Network-Based Power Systems, IEEE Trans. Ind. Electron. 65 (2) (2018) 1685-1694.
[23] A. Selivanov, E. Fridman, Distributed event-triggered control of diffusion semilinear PDEs, Automatica 68 (2016) 344-351.
[24] D. Silvestre, P. Rosa, J. P. Hespanha, C. Silvestre, Self-triggered and event-triggered set-valued observers, Inf. Sci. 426 (2) (2018) 61-86.
[25] J. Sun, Q. Yang, X. Liu, J. Chen, Event-triggered consensus for linear continuous-time multi-agent systems based on a predictor, Inf. Sci. DOI: 10.1016/j.ins.2018.03.028.
[26] P. Tabuada, Event-triggered real-time scheduling of stabilizing control tasks, IEEE Trans. Autom. Control 52 (9) (2007) 1680-1685.
[27] J. Wang, X.-M. Zhang, Y. Lin, X. Ge, Q.-L. Han, Event-triggered dissipative control for networked stochastic systems under non-uniform sampling, Inf. Sci. 447 (2018) 216–228.
[28] X. Wang, M. D. Lemmon, Event-triggering in distributed networked control systems, IEEE Trans. Autom. Control 56 (3) (2011) 586–601.
[29] S.-P. Xiao, H. Lian, H. Zeng, G. Chen, W. Zheng, Analysis on robust passivity of uncertain neural networks with time-varying delays via free-matrix-based integral inequality, Int. J. Control Autom. Syst. 15 (5) (2017) 2385-2394.

[30] Y. Yang, D. Yue, C. Dou, Output-based event-triggered schemes on leader-following consensus of a class of multi-agent systems with Lipschitz-type dynamics, Inf. Sci. DOI: 10.1016/j.ins.2018.02.020

[31] H. Yu, P. J. Antsaklis, Event-triggered real-time scheduling for stabilization of passive and output feedback passive systems, in: Proceedings of the American Control Conference (ACC), IEEE, 2011, pp. 1674–1679.

[32] D. Yue, E. Tian, Y. Zhang, C. Peng, Delay-distribution-dependent stability and stabilization of T-S fuzzy systems with probabilistic interval delay, IEEE Trans. Syst. Man Cyber. 39 (2) (2009) 503–516.

[33] D. Zhang, Q.-L. Han, X. Jia, Network-based output tracking control for T-S fuzzy systems using an event-triggered communication scheme, Fuzzy Sets Sys. 273 (2015) 26-48.

[34] B.-L. Zhang, Q.-L. Han, X.-M. Zhang, Event-triggered H_∞ reliable control for offshore structures in network environments, J. Sound Vib. 368 (2016) 1-21.

[35] B.-L. Zhang, Q.-L. Han, X.-M. Zhang, Recent advances in vibration control of offshore platforms, Nonlin. Dyn. 89 (2) (2017) 755-771.

[36] B.-L. Zhang, Q.-L. Han, X.-M. Zhang, X. Yu, Sliding mode control with mixed current and delayed states for offshore steel jacket platform, IEEE Trans. Control Syst. Technol. 22 (5) (2014) 1769-1783.

[37] H. Zhang, D. Yue, X. Yin, S. Hu, C. Dou, Finite-time distributed event-triggered consensus control for multi-agent systems, Inf. Sci. 339 (2016) 132–142.

[38] X.-M. Zhang, Q.-L. Han, Event-triggered dynamic output feedback control for networked control systems, IET Control Theory Appl. 8 (4) (2014) 226-234.

[39] X.-M. Zhang, Q.-L. Han, Novel delay-derivative-dependent stability criteria using new bounding techniques, Int. J. Robust Nonlin. Control 23 (13) (2013) 1419-1432.

[40] X.-M. Zhang, Q.-L. Han, Network-based H_∞ filtering using a logic jumping-like trigger, Automatica 49 (5) (2013) 1428-1435.

[41] X.-M. Zhang, Q.-L. Han, Event-triggered H_∞ control for a class of nonlinear networked control systems using novel integral inequalities, Int. J. Robust. Nonlinear Control 27 (4) (2017) 679–700.

[42] X.-M. Zhang, Q.-L. Han, B. -L. Zhang, An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems, IEEE Trans. Ind. Inform. 13 (1) (2017) 4–16.

[43] Z. Zhang, L. Zhang, F. Hao, L. Wang, Distributed event-triggered consensus for multi-agent systems with quantisation, Int. J. Control 88 (6) (2015) 1112–1122.

[44] K. Zhu, Y. Song, D. Ding, G. Wei, H. Liu, Robust MPC under event-triggered mechanism and Round-Robin Protocol: An average dwell-time approach, Inf. Sci. DOI: 10.1016/j.ins.2018.04.052.

[45] Z. Zuo, Q.-L. Han, B. Ning, X. Ge, X.-M. Zhang, An overview of recent advances in fixed-time cooperative control of multi-agent systems, IEEE Trans. Ind. Inform. DOI: 10.1109/TII.2018.2817248.